

$$\sin 3x - 2\sin 18x \sin x = 3\sqrt{2} - \cos 3x + 2\cos x$$

$$\sin 3x + \cos 3x = 2\sin 18x \sin x + 3\sqrt{2} + 2\cos x$$

$$\sqrt{2}(\sin 3x \cdot 1/\sqrt{2} + \cos 3x \cdot 1/\sqrt{2}) = \sqrt{4\sin^2 18x + 4}(\sin x \cdot 2\sin 18x/\sqrt{4\sin^2 18x + 4} + \cos x \cdot 2/\sqrt{4\sin^2 18x + 4}) + 3\sqrt{2}$$

$$\sin a = 1/\sqrt{2}$$

$$\cos a = 1/\sqrt{2}$$

$$\sin b = 2\sin 18x/\sqrt{4\sin^2 18x + 4}$$

$$\cos b = 2/\sqrt{4\sin^2 18x + 4}$$

$$\sqrt{2}(\sin 3x \cos a + \cos 3x \sin a) = \sqrt{4\sin^2 18x + 4}(\sin x \cos b + \cos x \sin b) + 3\sqrt{2}$$

$$\sqrt{2}(\sin(3x+a)) = \sqrt{4\sin^2 18x + 4}(\sin(x+b)) + 3\sqrt{2}$$

$$\text{Max}(\sqrt{2}(\sin(3x+a))) = \sqrt{2}$$

$$\text{max}(\sqrt{4\sin^2 18x + 4}(\sin(x+b)) + 3\sqrt{2}) = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$$

$$\text{min}(\sqrt{4\sin^2 18x + 4}(\sin(x+b)) + 3\sqrt{2}) = -2\sqrt{2} + 3\sqrt{2} = \sqrt{2}$$

$$\sqrt{2} = \sqrt{2}$$

$$\sqrt{2}(\sin(3x+a)) = \sqrt{2}$$

$$a = \pi/4$$

$$3x = \pi/2 + 2\pi k - \pi/4$$

$$3x = \pi/4 + 2\pi k$$

$$x = \pi/12 + 2\pi k/3$$

$$x_1 = \pi/12$$

$$x_2 = 3\pi/4$$

$$x_3 = 17\pi/12$$

$$f(x) = 2\sin 18x \sin x + 3\sqrt{2} + 2\cos x$$

$$f(x_1) = 2\sin(9\pi/2) \sin(\pi/4) + 3\sqrt{2} + 2\cos(\pi/4)$$

$$\sqrt{2} + 3\sqrt{2} + \sqrt{2}/2 = 4\sqrt{2} + \sqrt{2}/2 - \text{не подходит}$$

$$f(x_2) = 2\sin(54\pi/4) \sin(3\pi/4) + 3\sqrt{2} + 2\cos(3\pi/4)$$

$$2\sqrt{2}/2 + 3\sqrt{2} - 2\sqrt{2}/2 = 3\sqrt{2} - \text{не подходит}$$

$$f(x_3) = 2\sin(\pi/2) \sin(17\pi/12) + 3\sqrt{2} + 2\cos(17\pi/12) - \text{не подходит}$$

Ответ: нет решений

